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A CONDITIONED LIMIT LAW RESULT FOR JUMPS IN THE SEQUENCE OF PAR--ETC(U)

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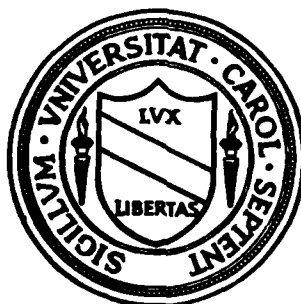
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Department of Statistics  
University of North Carolina  
Chapel Hill, North Carolina



A CONDITIONED LIMIT LAW RESULT FOR JUMPS IN THE  
SEQUENCE OF PARTIAL MAXIMA OF A STATIONARY GAUSSIAN SEQUENCE

by  
William P. McCormick

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A Conditioned Limit Law Result For Jumps In The  
Sequence of Partial Maxima of a Stationary Gaussian  
Sequence

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Keywords: Gaussian sequences, maxima, weak convergence, point processes.

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Conditional of a jump occurring, the limiting distribution for the size of the jump in the partial maxima sequence for a class of stationary Gaussian sequences is derived. It is shown that the limiting distribution is exponential with mean  $\sqrt{1-\gamma}$  where  $\gamma$  equals the atom at zero of the spectral distribution function associated with the correlation function of the sequence. A generalization of this result to include the entire jump sequence subsequent to the jump conditioned to occur is also presented.

This research was done in part while the author was at the Center for Stochastic Processes at Chapel Hill. The author wants to express his gratitude to Chapel Hill for the financial assistance and hospitality shown to him.

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# 1. INTRODUCTION

Let  $\{X_k, k \geq 0\}$  be a stationary sequence of standard normal random variables with correlation function  $r$ . In the main result of this paper we will require the correlation function to satisfy the condition that there exists some  $0 < \alpha < 1$  such that

$$(1.1) \quad \sup_{[n^\alpha] \leq k \leq n} |r_k - r_n| = o\left(\frac{1}{\ln n}\right)$$

and that  $r$  is not identically one.

Condition (1.1) is of course not a mixing condition. In fact it follows from (1.1) that  $r_n \rightarrow \gamma$  as  $n \rightarrow \infty$  where  $\gamma$  is the atom at zero of the spectral distribution function associated with  $r$ , (2.6) of [3]. Note  $0 \leq \gamma < 1$  since  $r$  is assumed to be not identically 1 and so by stationarity  $\sup_n |r_n| < 1$ .

Furthermore we have the simple but useful representation of the first  $n+1$  terms of the sequence given by

$$(1.2) \quad X_k = \sigma_n(k) Y_k^n + \bar{X}_n, \quad 0 \leq k \leq n$$

where  $Y_k^n = \frac{X_k - \bar{X}_n}{\sigma_n(k)}$ ,  $\bar{X}_n = \frac{1}{n+1} \sum_{k=0}^n X_k$ , and  $\sigma_n^2(k) = E(X_k - \bar{X}_n)^2$ .

Under condition (1.1) it follows [3] that  $\rho_n(i, j) = E Y_i^n Y_j^n$  satisfies

$$\max_{0 \leq i \leq j \leq n} \left| \rho_n(i, j) - \frac{r(i-j) - r(n)}{1 - r(n)} \right| = o\left(\frac{1}{\ln n}\right)$$

Thus we see that condition (1.1) imposes a mixing condition on the  $Y_k^n$  variables and that  $\bar{X}_n$  referred to as a binding variable is responsible for any strong dependence in the original sequence. Gaussian processes with this type of structure have been considered in a number of

papers e.g. [3], [5] and it is the presence of such a binding variable which effects departures from the usual asymptotic results in the strongly dependent case.

The problem under consideration in this paper is that of jumps in the partial maxima sequence. Denoting  $M_m = \max_{0 \leq k \leq m} X_k$  and  $M_{m,n}^* = \max_{0 \leq k \leq m} Y_k^n$  we see that a jump occurs at time  $m = n$  in the  $M_m$  sequence if and only if a jump occurs at time  $m = n$  in the  $M_{m,n}^*$  sequence. Consequently the binding variable does not affect the occurrence of jumps and we expect that under (1.1) the limiting behavior for the jumps in the dependent case should agree with the corresponding result for independent variables. That this is indeed the case is demonstrated in section 2.

## 2. CONDITIONED LIMIT LAW FOR JUMPS

Theorem 2.1 Let  $\{X_k, k \geq 0\}$  be a stationary sequence of standard normal random variables with correlation function  $r_n = E X_0 X_n$  not identically one and satisfying

$$(2.1) \quad \sup_{[n^\alpha] \leq k \leq n} |r_k - r_n| = o\left(\frac{1}{\ln n}\right) \text{ for some fixed } 0 < \alpha < 1.$$

Then with  $M_n = \max_{0 \leq k \leq n} X_k$  and  $c_n = \sqrt{2 \ln n}$ , we have

$$(2.2) \quad \lim_{n \rightarrow \infty} P\{c_n(M_n - M_{n-1}) > x \mid M_n - M_{n-1} > 0\} = e^{-\frac{x}{\sqrt{1-\gamma}}}, \quad x > 0$$

where  $\gamma$  equals the atom at zero of the spectral distribution function associated with  $r$ .

Before proceeding to the proof of Theorem 2.1 it will be of interest to have a uniform bound on the tail of the distribution for the normalized maxima. This is provided to us by the following result of Mittal, [4].

Lemma 2.1 (Mittal). For a stationary sequence of standard normal random variables  $\{X_k, k \geq 0\}$  with correlation function  $r_n = E X_0 X_n$  satisfying  $r_n \ln n = o(1)$ ,

$$e^{tA^2} P\{c_n(M_n - b_n) \leq -A\} = o(1) \text{ as } A \rightarrow \infty$$

uniformly in  $n$  for all  $t$  in some neighborhood of zero where  $b_n = c_n - \frac{\ln(4\pi \ln n)}{2 c_n}$ .

A check of the proof of this lemma reveals that it may be stated in the following form which is suitable to our purposes.

Lemma 2.2 Let  $\{X_{k,n}\}$ ,  $k = 0, \dots, n$ ,  $n = 1, 2, \dots$  be a triangular array of standard normal random variables. Then setting

$$r_n(i,j) = E X_{i,n} X_{j,n}, M_n = \max_{0 \leq k \leq n} X_{k,n} \text{ and } \delta_n(x) = \sup_{|i-j| \geq x} |r_n(i,j)|$$

we have

$$e^{tA^2} P\{c_n(M_n - b_n) \leq -A\} = o(1) \text{ as } A \rightarrow \infty$$

uniformly in  $n$  for all  $t$  in a neighborhood of zero provided

$$(i) \quad \lim_{n \rightarrow \infty} \delta_n(1) < 1$$

(2.3)

$$(ii) \quad \delta_n(n^\alpha) \ln n = o(1) \text{ for some fixed } 0 < \alpha < 1.$$

In order to prove the conditioned limit law result we need to determine the rate at which the probability of the conditioning event goes to zero and to establish an unconditional limit law result.

Lemma 2.3 Under the conditions of Theorem 2.1 we have

$$(2.4) \quad \lim_{n \rightarrow \infty} n P\{M_n - M_{n-1} > o\} = 1$$

Proof. Since  $(X_0, X_1, \dots, X_n) \stackrel{D}{=} (X_n, X_{n-1}, \dots, X_0)$  we have

$$\begin{aligned}
P\{M_n > M_{n-1}\} &= P\{M_n > \max_{1 \leq k \leq n} Y_k\} \\
&= P\{c_n(Y_0^n - b_n) > c_n \frac{\sigma_n(k)}{\sigma_n(o)} Y_k^n - b_n, 1 \leq k \leq n\} \\
&= \int_{-\infty}^{\infty} P\{c_n \frac{\sigma_n(k)}{\sigma_n(o)} Y_k^n - b_n < y, 1 \leq k \leq n \mid Y_0^n = b_n + \frac{y}{c_n}\}
\end{aligned}$$

$$\begin{aligned}
&\phi(b_n + \frac{y}{c_n}) \frac{dy}{c_n} \\
&= \frac{\phi(b_n)}{c_n} \int_{-\infty}^{\infty} P\{W_k^n < (\frac{1 - \rho_n(o,k)}{1 + \rho_n(o,k)})^{1/2} (b_n + \frac{y}{c_n}) + \theta_n(k,y), \\
&\quad 1 \leq k \leq n\} e^{-\frac{b_n}{c_n} y - \frac{y^2}{4 \ln n}} dy
\end{aligned}$$

$$\text{where } \theta_n(k,y) = (1 - \rho_n^2(o,k))^{-1/2} (\frac{\sigma_n(o)}{\sigma_n(k)} - 1) (b_n + \frac{y}{c_n}),$$

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \text{ and where the } W_k^n \text{ are standard normal random}$$

variables with correlation function

$$(2.5) \quad \zeta_n(i,j) = E W_i^n W_j^n = \frac{\rho_n(i,j) - \rho_n(o,i) \rho_n(o,j)}{(1 - \rho_n(o,i))^{1/2} (1 - \rho_n(o,j))^{1/2}}$$

Therefore since  $\frac{\phi(b_n)}{c_n} \sim \frac{1}{n}$  it suffices to show that

$$(2.6) \quad \lim_{n \rightarrow \infty} \int_{-\infty}^{\infty} P\{W_k^n \leq (\frac{1 - \rho_n(o,k)}{1 + \rho_n(o,k)})^{1/2} (b_n + \frac{y}{c_n}) + \theta_n(k,y), 1 \leq k \leq n\}$$

$$\exp \left[ -\frac{b_n}{c_n} y - \frac{y^2}{4 \ln n} \right] dy = 1$$



The statement above is verified via dominated convergence and Lemma 2.4 below. Denoting the integrand at (2.6) by  $f_n(y)$  we have for any fixed positive number  $K$  that  $f_n(y) < e^{K-y/2}$ ,  $-K \leq y < \infty$  for all  $n$  sufficiently large. To obtain an integrable uniform bound for the  $f_n$  when  $-\infty < y < -K$  we appeal to Lemma 2.2.

First note that by (2.8) of [3]  $\max_{1 \leq k \leq n} |\sigma_n^2(k) - (1 - r_n)| = o(\frac{1}{\ln n})$  so that with  $\alpha$  chosen such that condition (2.1) holds, we have for all  $n$  large

$$\begin{aligned} P\{W_k^n \leq \frac{1 - \rho_n(o,k)^{1/2}}{1 + \rho_n(o,k)} (b_n + \frac{y}{c_n}) + \theta_n(k,y), 1 \leq k \leq n\} \\ \leq P\{W_k^n \leq b_n + \frac{y}{2c_n}, [n^\alpha] \leq k \leq n\} \\ (2.7) \quad \leq P\{\max_{0 \leq k \leq n - [n^\alpha]} W_k^n + [n^\alpha] \leq b_{n - [n^\alpha]} + \frac{y}{4c_{n - [n^\alpha]}}\} \end{aligned}$$

Next consider the triangular array  $V_{k, n - [n^\alpha]} = W_{k + [n^\alpha]}^n$ ,  $0 \leq k \leq n - [n^\alpha]$ . It is easily checked that condition (2.3 i) is satisfied for the above array and that (2.3 ii) holds for any fixed  $\alpha'$  with  $\alpha < \alpha' < 1$ . Therefore by (2.7) and Lemma 2.2 we have that for all  $n$  sufficiently large

$$(2.8) \quad f_n(y) \leq \begin{cases} e^{K-y/2}, & -K \leq y < \infty \\ e^{-ty^2-y}, & -\infty < y < -K \end{cases}$$

for some positive  $t$  and  $K$  not depending on  $n$ . Therefore assuming (2.9) we have by dominated convergence applicable by (2.8) that the limit at (2.6) exists and equals  $\int_{-\infty}^{\infty} e^{-e^{-y}} e^{-y} dy = 1$  completing the proof of Lemma 2.3.

Lemma 2.4 Let  $\{W_k^n\}$   $k = 1, \dots, n$ ;  $n=1, \dots$  be a triangular array of standard normal random variables with covariance function  $\zeta_n(i,j) = E W_k^n W_j^n$  given by (2.5). Then

$$(2.9) \quad \lim_{n \rightarrow \infty} P\{W_k^n \leq \frac{1 - \rho_n(o,k)}{1 + \rho_n(o,k)}^{1/2} (b_n + \frac{y}{c_n}) + \theta_n(k,y), 1 \leq k \leq n\} = e^{-e^{-y}}, \quad -\infty < y < \infty$$

Proof. A simple argument shows that for any fixed  $\varepsilon > 0$ , we have for all  $n$  sufficiently large and for  $0 < \alpha < 1$  chosen so that condition (2.1) applies that

$$\begin{aligned} & P\{W_k^n \leq b_n + \frac{y - \varepsilon}{c_n}, [n^\alpha] < k \leq n\} - \varepsilon \\ & \leq P\{W_k^n \leq \frac{1 - \rho_n(o,k)}{1 + \rho_n(o,k)}^{1/2} (b_n + \frac{y}{c_n}) + \theta_n(k,y), 1 \leq k \leq n\} \\ & \leq P\{W_k^n \leq b_n + \frac{y + \varepsilon}{c_n}, [n^\alpha] < k \leq n\} \end{aligned}$$

Therefore it suffices to show

$$(2.10) \quad \lim_{n \rightarrow \infty} P\{W_k^n \leq b_n + \frac{y}{c_n}, [n^\alpha] < k \leq n\} = e^{-e^{-y}}$$

By Berman's lemma [1] we have

$$\begin{aligned}
& |P\{W_k^n \leq b_n + \frac{y}{c_n}, [n^\alpha] < k \leq n\} - \phi^{n-[n^\alpha]}(b_n + \frac{y}{c_n})| \\
& \leq (\text{CONST.}) \sum_{[n^\alpha] < i < j \leq n} |\zeta_n(i, j)| \exp\{-\frac{(b_n + \frac{y}{c_n})^2}{1 + |\zeta_n(i, j)|}\} \\
& \leq (\text{CONST.}) \sum_{1 \leq i < j \leq n} |\rho_n(i, j)| \exp\{-\frac{(b_n + \frac{y}{c_n})^2}{1 + |\rho_n(i, j)|}\} + o(1) \\
& = o(1)
\end{aligned}$$

where the last summation above is  $o(1)$  by the work in [3]. Thus (2.10) holds since it is true for independent  $W_k^n$ .

Proof of Theorem 2.1

$$\begin{aligned}
& P\{c_n(M_n - M_{n-1}) > x \mid M_n - M_{n-1} > 0\} \\
& = (1+o(1)) \frac{\int_{-\infty}^{\infty} P\{W_k^n \leq \frac{1-\rho_n(o, k)}{1+\rho_n(o, k)} (b_n + \frac{y-x/\sigma_n(o)}{c_n}) + \theta_n(k, y-x/\sigma_n(o)), 1 \leq k \leq n\} dy}{nP\{M_n - M_{n-1} > 0\}} \\
& = (1+o(1)) e^{-\frac{b_n}{c_n} y - \frac{y^2}{4 \ln n}} \int_{-\infty}^{\infty} P\{W_k^n \leq \frac{1-\rho_n(o, k)}{1+\rho_n(o, k)} (b_n + \frac{y}{c_n}) + \theta_n(k, y), 1 \leq k \leq n\} e^{\frac{x}{\sigma_n(o)} y} dy \\
& \quad \cdot e^{-\frac{(y + \frac{x}{\sigma_n(o)})^2}{4 \ln n}} dy \\
& \rightarrow e^{-\frac{x}{\sqrt{1-\gamma}}} \text{ as } n \rightarrow \infty.
\end{aligned}$$

### 3. CONVERGENCE OF POINT PROCESSES

A natural generalization of the result in section 2 is to consider the asymptotic behavior of the sequence of jump sizes conditional on a jump occurring at time  $n$ . A result of this type is conveniently formulated as a convergence result for point processes.

For each  $n$  fixed define jump times  $\{\tau_{n,k}; k \geq 0\}$  by

$$\tau_{n,0} = 0$$

$$\tau_{n,k} = \inf\{i > \tau_{n,k-1} : M_{n+i} > M_{n+\tau_{n,k-1}}\}, \quad k \geq 1$$

and jump sizes  $\{J_{n,k}; k \geq 0\}$  by

$$J_{n,0} = M_n - M_{n-1}$$

$$J_{n,k} = M_{n+\tau_{n,k}} - M_{n+\tau_{n,k-1}}, \quad k \geq 1.$$

Next define point processes  $\eta_n$  where for  $B \in \mathcal{B}(R_+)$

$$\eta_n(B) = \sum_{k=0}^{\infty} I_B\left(c_n \sum_{i=0}^k J_{n,i}\right)$$

Under the conditions of Theorem 2.1 we show that conditional on a jump occurring at time  $n$  the  $\eta_n$  converge in distribution with respect to the vague topology to homogeneous Poisson random measure with intensity  $(1-\gamma)^{-1/2}$ . Or equivalently conditional on a jump at time  $n$  for  $n$  sufficiently large the jumps  $J_{n,k}; k \geq 0$  are approximately i.i.d. exponential with mean  $\sqrt{1-\gamma}$ .

Define point processes  $M_n$  on  $R_+ \times R$  by

$$M_n(B) = \sum_{k=0}^{\infty} I_B\left(\frac{k}{n}, c_n(X_{n+k} - M_{n-1})\right)$$

Let  $M$  be doubly stochastic Poisson random measure with mean measure  $m$  given by

$$m([0, t] \times (x, \infty)) = t \exp\{-(1-\gamma)^{-\frac{1}{2}} x + \Lambda\}$$

where  $\Lambda \sim (1+e^{-X}) \exp\{-e^{-X}\}$ .

Let  $\delta_{(0, x)}$  denote the degenerate measure at  $(0, x)$  and  $X \sim 1 - e^{-\frac{x}{\sqrt{1-\gamma}}}$ ,  $x > 0$  be independent of  $M$ .

Lemma 3.1. Under the hypothesis of Theorem 2.1 we have conditional on

$$(M_n - M_{n-1} > 0).$$

$$(3.1) \quad M_n \Rightarrow M + \delta_{(0, X)}$$

Proof. To show (3.1) it suffices to show Theorem 4.7 of Kallenberg [2] that

$$(i) \quad \lim_{n \rightarrow \infty} E[M_n(U) | M_n - M_{n-1} > 0] < E[M(U) + \delta_{(0, X)}^{(U)}]$$

$$(ii) \quad \lim_{n \rightarrow \infty} P\{M_n(U) = 0 | M_n - M_{n-1} > 0\} = P\{M(U) + \delta_{(0, X)}^{(U)} = 0\}$$

where  $U = \bigcup_{i=1}^{\infty} I_i \times B_i$  is a disjoint union of bounded rectangles. Since the proof of (3.2) is routine, we will not present the proof which is long and very detailed.

Let  $\delta(B) = \sum_{k=0}^{\infty} l_B(t_k, x_k)$  be a simple integer valued Radon measure on  $\mathbb{R}_+ \times \mathbb{R}$  with points  $\{(t_k, x_k), k \geq 0\}$ . We also call  $\delta$  a point system. Define a map  $g$  on the space of point systems to  $D[0, \infty)$  by

$$g\delta(t) = \max\{x_k : t_k \leq t\}.$$

Then from the space of point systems endowed with the vague topology to  $D[0, \infty)$  with the Skorohod  $J_1$  topology,  $g$  is continuous, [7]. Let

$$Y_n(t) = gM_n(t) = c_n \left( \max_{0 \leq k \leq [nt]} X_{n+k} - M_{n-1} \right), \quad t \geq 0$$

and

$$Y(t) = (1-\gamma)^{\frac{1}{2}} \max(X, Z(t) - \Lambda), \quad t \geq 0.$$

where  $X$  and  $\Lambda$  are independent and independent of the process  $\{Z(t), t \geq 0\}$ .  $X \sim 1 - e^{-x}$ ,  $x > 0$ ,  $\Lambda \sim (1 + e^{-x}) \exp(-e^{-x})$  and  $\{Z(t), t \geq 0\}$  is an extremal  $\exp(-e^{-x})$  process, that is for  $0 < t_1 < t_2 < \dots < t_n$

$$P\{Z(t_1) \leq x_1, Z(t_2) \leq x_2, \dots, Z(t_n) \leq x_n\} \\ = F^{t_1}(\min\{x_1, \dots, x_n\}) F^{t_2 - t_1}(\min\{x_2, \dots, x_n\}) \dots F^{t_n - t_{n-1}}(x_n)$$

with  $F(x) = \exp(-e^{-x})$ .

Lemma 3.2. Under the hypothesis of Theorem 2.1 we have conditional on

$$(M_n - M_{n-1} > 0)$$

$$(3.3) \quad Y_n(\cdot) \Rightarrow Y(\cdot)$$

where  $\Rightarrow$  denotes convergence in distribution with respect to the Skorohod  $J_1$  topology on  $D[0, \infty)$ .

Proof. It is easily checked that  $g^{(M+\delta)_{(0,X)}}(\cdot) \stackrel{d}{=} Y(\cdot)$ . Therefore (3.3) follows from Lemma 3.1 and the continuous mapping theorem.

Conditional on  $\Lambda = \lambda$  the process  $Y(t)$  is a pure jump Markov process. Its jumping measure  $K_\lambda(x, B)$ , which denotes the probability that the process after leaving state  $x$  enters set  $B$ , is calculated as in [6] and is given by

$$K_\lambda(x, dy) = -I_{[y > x]} \frac{Q_\lambda(dy)}{Q_\lambda(x)}$$

where  $Q_\lambda(y) = \exp\{-\frac{y}{\sqrt{1-\gamma}} + \lambda\}$ .

Set  $Y(t) = \xi_k, \tau_k \leq t < \tau_{k+1}$

where  $\tau_0 = 0$  and  $\xi_0 = (1-\gamma)^{1/2} X$ ,  $X \sim 1 - e^{-x}$ ,  $x > 0$ .

Let  $h$  denote the map which extracts the states from the pure jump functions in  $D[0, \infty)$ . Then  $hY_n = \eta_n$  and  $hY = \{\xi_k; k \geq 0\}$ .

Theorem 3.1. Assume the hypothesis of Theorem 2.1. Then conditional on  $(M_n - M_{n-1} > 0)$   $\eta_n \Rightarrow \eta$  where  $\eta$  is homogeneous Poisson random measure with intensity  $(1-\gamma)^{-\frac{1}{2}}$ .

Proof. Since the map  $h$  defined above is continuous, it remains only to identify the limit  $hY = \{\xi_k; k \geq 0\}$  as homogeneous Poisson random measure with intensity  $(1-\gamma)^{-\frac{1}{2}}$ . But for  $0 < x_0 < x_1 < \dots < x_k$  we have

$$\begin{aligned} & P\{\xi_0 \in dx_0, \xi_1 \in dx_1, \dots, \xi_k \in dx_k\} \\ &= E[P\{\xi_0 \in dx_0, \xi_1 \in dx_1, \dots, \xi_k \in dx_k | \Lambda = \lambda\}] \\ &= E\left[\frac{1}{\sqrt{1-\gamma}} e^{-\frac{x_0}{\sqrt{1-\gamma}}} \prod_{i=1}^k K_\lambda(x_{i-1}, dx_i)\right] \\ &= \frac{1}{\sqrt{1-\gamma}} e^{-\frac{x_0}{\sqrt{1-\gamma}}} \prod_{i=1}^k \left(\frac{1}{\sqrt{1-\gamma}} e^{-\frac{x_i - x_{i-1}}{\sqrt{1-\gamma}}}\right) dx_0 \dots dx_k \\ &= \frac{1}{\frac{k+1}{2}} e^{-\frac{x_k}{\sqrt{1-\gamma}}} dx_0 \dots dx_k. \\ & \quad (1-\gamma)^{\frac{1}{2}} \end{aligned}$$

Hence the result follows.

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18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Gaussian sequences; maxima; weak convergence; joint processes.  <i>square root of (1 - gamma)</i>		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  Conditional of a jump occurring, the limiting distribution for the size of the jump in the partial maxima sequence for a class of stationary Gaussian sequences is derived. It is shown that the limiting distribution is exponential with mean <del>1</del> where $\gamma$ equals the atom at zero of the spectral distribution function associated with the correlation function of the sequence. A generalization of this result to include the entire jump sequence subsequent to the jump condi- tioned to occur is also presented.		